# NORMANHURST BOYS HIGH SCHOOL

# MATHEMATICS ADVANCED

2023 Year 12 Course Assessment Task 4 (Trial HSC Examination)
Wednesday, 16 August 2023

**General instructions**

- Working time – 3 hours. (plus 10 minutes reading time)
- Write using blue or black pen. Where diagrams are to be sketched, these may be done in pencil.
- NESA approved calculators may be used.
- Attempt **all** questions.

**SECTION I**

- Mark your answers on the answer grid provided (on page 29)

**SECTION II**

- All necessary working should be shown in every question. Marks may be deducted for illegible or incomplete working.

**NESA STUDENT #:** ..........................

**Class** (please ✔)

- ◯ 12MAA.1 – Mr Lam
- ◯ 12MAX.2 – Ms Lee
- ◯ 12MAX.1 – Ms C. Kim
- ◯ 12MAX.3 – Ms J. Kim

Marker's use only.

| QUESTION | 1-10 | 11-14 | 15-17 | 18-19 | 20-21 | 22-24 | 25-26 | 27-28 | 29-30 | Total |
|---|---|---|---|---|---|---|---|---|---|---|
| MARKS | /10 | /15 | /13 | /9 | /7 | /16 | /8 | /11 | /11 | /100 |

# Section I

**10 marks**
**Attempt Question 1 to 10**
**Allow approximately 15 minutes for this section**

Mark your answers on the answer grid provided (labelled as page 29).

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**Questions** **Marks**

**1.** Simplify $\sqrt{x^2+2x+1}-\sqrt{x^2-2x+1}$ **1**

(A) $-2$

(B) $2$

(C) $2x$

(D) $2x+2$

**2.** Using the table below, which of the following is an expression for $E\left(X^2\right)$? **1**

| $x$ | 10 | 20 | 30 | 40 | 50 |
|---|---|---|---|---|---|
| $P(X=x)$ | $1-3a$ | $a$ | $1-9a$ | $1-10a$ | $a$ |

(A) $10^2(1-3a)+20^2a+30^2(1-9a)+40^2(1-10a)+50^2a$

(B) $10(1-3a)^2+20a^2+30(1-9a)^2+40(1-10a)^2+50a^2$

(C) $10^2(1-3a)^2+20^2a^2+30^2(1-9a)^2+40^2(1-10a)^2+50^2a^2$

(D) $\left[10(1-3a)+20a+30(1-9a)+40(1-10a)+50a\right]^2$

**3.** Which of the following represents the solutions for the equation $2^{2x}-5(2^x)+4=0$? **1**

(A) $x=0$ or $x=1$

(B) $x=0$ or $x=2$

(C) $x=\log_2 1$ or $x=\log_2 2$

(D) $x=1$ or $x=4$

**4.** What is the period of the function $f(x)=-3\sin\left(\dfrac{\pi x}{5}\right)$? **1**

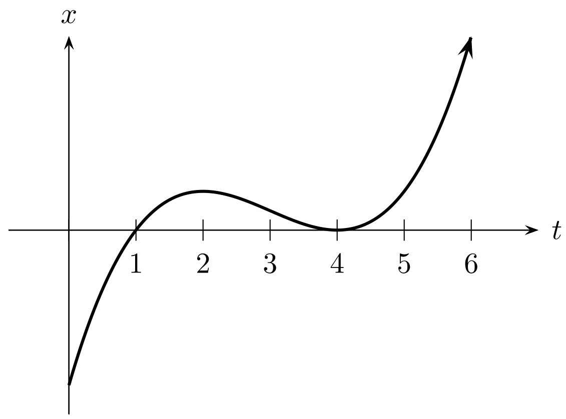
(A) 5

(B) $5\pi$

(C) 10

(D) $10\pi$

**5.** The displacement, $x$ metres, from the origin of a particle moving in a straight line at any time, $t$ seconds, is shown in the graph below. **1**




When was the particle at rest?

(A) $t = 0$

(B) $t = 1$ and $t = 4$

(C) $t = 2$ and $t = 4$

(D) $t = 1$, $t = 2$ and $t = 4$

**6.** Evaluate $\lim_{x \to 3} \dfrac{x^2 - 2x - 3}{x - 3}$ **1**

(A) 0

(B) undefined

(C) 4

(D) 1

**7.** Which of the following represents the domain and range of the function **1**

$$f(x) = \sqrt{4 - x^2}$$

(A) Domain: $-2 \leq x \leq 2$, Range: $0 \leq y \leq 2$

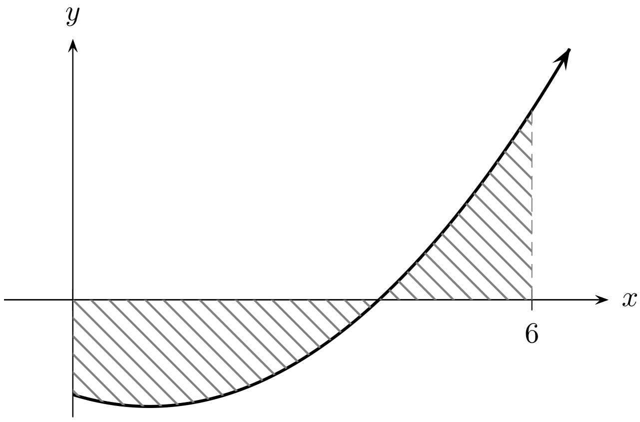
(B) Domain: $-2 \leq x \leq 2$, Range: $-2 \leq y \leq 2$

(C) Domain: $0 \leq x \leq 2$, Range: $-4 \leq y \leq 4$

(D) Domain: $0 \leq x \leq 2$, Range: $0 \leq y \leq 4$

**Examination continues overleaf...**

**8.** The diagram below shows the graph of $y = x^2 - 2x - 8$. **1**




What is the correct expression for the area bounded by the $x$-axis and the curve $y = x^2 - 2x - 8$ between $0 \le x \le 6$?

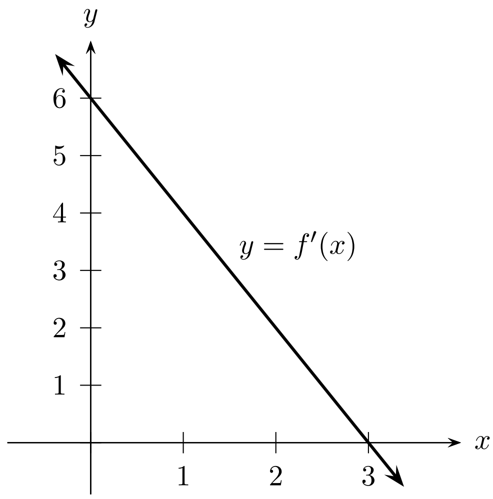
(A) $A = \int_0^5 \left(x^2 - 2x - 8\right)\, dx + \left| \int_5^6 \left(x^2 - 2x - 8\right)\, dx \right|$

(B) $A = \int_0^4 \left(x^2 - 2x - 8\right)\, dx + \left| \int_4^6 \left(x^2 - 2x - 8\right)\, dx \right|$

(C) $A = \left| \int_0^5 \left(x^2 - 2x - 8\right)\, dx \right| + \int_5^6 \left(x^2 - 2x - 8\right)\, dx$

(D) $A = \left| \int_0^4 \left(x^2 - 2x - 8\right)\, dx \right| + \int_4^6 \left(x^2 - 2x - 8\right)\, dx$

**9.** The graph of $y = f'(x)$ is shown below. **1**




The curve $y = f(x)$ has a maximum value of 20. Which of the following is the equation of $f(x)$?

(A) $y = x^2 - 6x + 11$

(B) $y = x^2 - 6x + 20$

(C) $y = -x^2 + 6x + 20$

(D) $y = -x^2 + 6x + 11$

**10.** Which of the following is equal to $\log_c(a) + \log_a(b) + \log_b(c)$? **1**

(A) $\dfrac{1}{\log_c(a)} + \dfrac{1}{\log_a(b)} + \dfrac{1}{\log_b(c)}$

(B) $\dfrac{1}{\log_a(c)} + \dfrac{1}{\log_b(a)} + \dfrac{1}{\log_c(b)}$

(C) $-\dfrac{1}{\log_a(b)} - \dfrac{1}{\log_b(c)} - \dfrac{1}{\log_c(a)}$

(D) $\dfrac{1}{\log_a(a)} + \dfrac{1}{\log_b(b)} + \dfrac{1}{\log_c(c)}$

**Examination continues overleaf…**

## Answer sheet for Section I

Mark answers to Section I by fully blackening the correct circle, e.g "●"

**NESA STUDENT #:** ..............................................

**Class** (please ✔)

○ 12MAA.1 – Mr Lam ○ 12MAX.2 – Ms Lee

○ 12MAX.1 – Ms C. Kim ○ 12MAX.3 – Ms J. Kim

**Directions for multiple choice answers**

- Read each question and its suggested answers.
- Select the alternative Ⓐ, Ⓑ, Ⓒ, or Ⓓ that best answers the question.
- Mark only one circle per question. There is only *one* correct choice per question.
- Fill in the response circle completely, using blue or black pen, e.g.

  Ⓐ Ⓑ ● Ⓓ

- If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

  Ⓐ Ⓑ ⊗ ●

- If you continue to change your mind, write the word `correct` and clearly indicate your final choice with an arrow as shown below:

  Ⓐ Ⓑ ⊗ ⊗ ← correct

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1** – | Ⓐ | Ⓑ | Ⓒ | Ⓓ | **6** – | Ⓐ | Ⓑ | Ⓒ | Ⓓ |
| **2** – | Ⓐ | Ⓑ | Ⓒ | Ⓓ | **7** – | Ⓐ | Ⓑ | Ⓒ | Ⓓ |
| **3** – | Ⓐ | Ⓑ | Ⓒ | Ⓓ | **8** – | Ⓐ | Ⓑ | Ⓒ | Ⓓ |
| **4** – | Ⓐ | Ⓑ | Ⓒ | Ⓓ | **9** – | Ⓐ | Ⓑ | Ⓒ | Ⓓ |
| **5** – | Ⓐ | Ⓑ | Ⓒ | Ⓓ | **10** – | Ⓐ | Ⓑ | Ⓒ | Ⓓ |

# Section II

**90 marks**
**Attempt Question 11 to 30**
**Allow approximately 2 hours and 45 minutes for this section**

Write your answers in the space provided.

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**Question 11** (5 marks)

Sam and Jammy are planning a fund-raising dance. They can hire a hall for \$400 and a band for \$300. Refreshments will cost them \$12 per person.

(a) Write a formula for the cost ($\$C$) of running the dance for $x$ people. **1**

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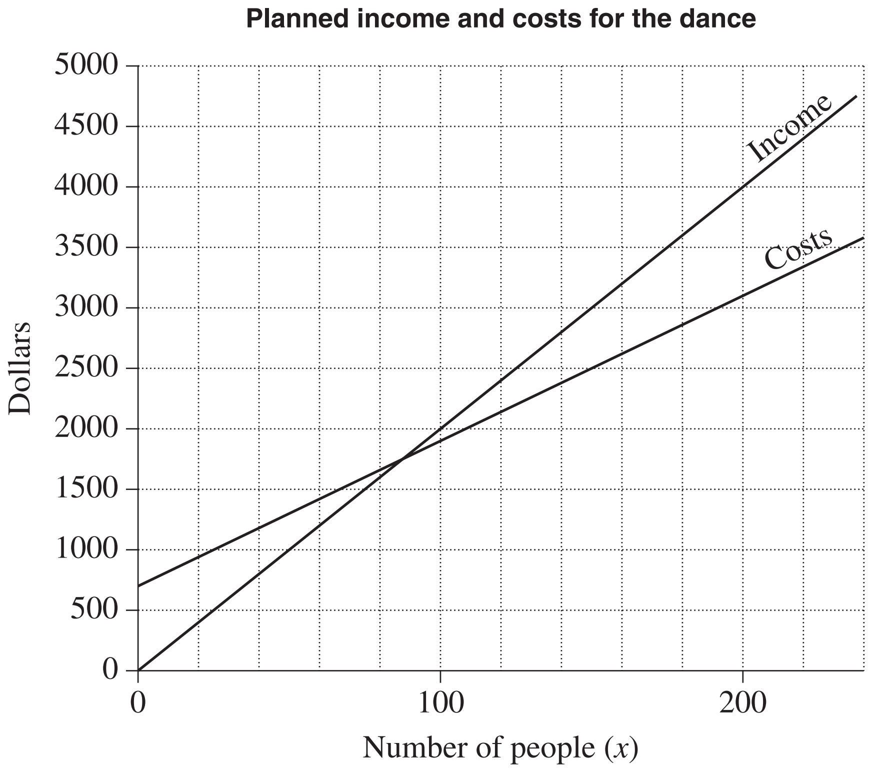
The graph shows planned income and costs when the ticket price is \$20.




(b) Estimate the minimum number of people needed at the dance to cover the costs. **1**

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(c) How much profit will be made if 150 people attend the dance? **1**

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(d) Sam and Jammy plan to sell 200 tickets. They want to make a profit of \$1500. What should be the price of a ticket, assuming all 200 tickets will be sold? **2**

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**Question 12** (4 marks)

The graph of the function $f(x)$ is obtained from the graph of the function **4**

$$g(x) = 3\cos\left(x - \frac{\pi}{6}\right)$$

by applying the following transformations.

1. A dilation of a factor of $\frac{1}{2}$ from the $x$-axis
2. A reflection in the $y$-axis
3. A translation of $\frac{\pi}{6}$ units in the negative $x$ direction
4. A translation of 4 units in the negative $y$ direction

Find the rule of $f(x)$.

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**Examination continues overleaf**...

**Question 13** (3 marks)

A sector has radius length of 20 cm and the angle subtended at the centre is $50°$. The radius of this sector increased by 25% and its angle at the centre is decreased by $k\%$. **3**

If the area of the sector remains unchanged find the value of $k$.

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**Question 14** (3 marks)

Solve for $x$: **3**

$$2\ln(x+2) - \ln x = \ln(2x+1) \qquad \text{where } x > 0$$

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**Question 15** (3 marks)

Find the equation of the normal to the curve $y = 2(5x-4)^4$ at $x = 1$. **3**
Express your answer in general form $ax + by + c = 0$.

**Examination continues overleaf...**

**Question 16** (3 marks)

A rational function $f(x)$ has the following properties: **3**

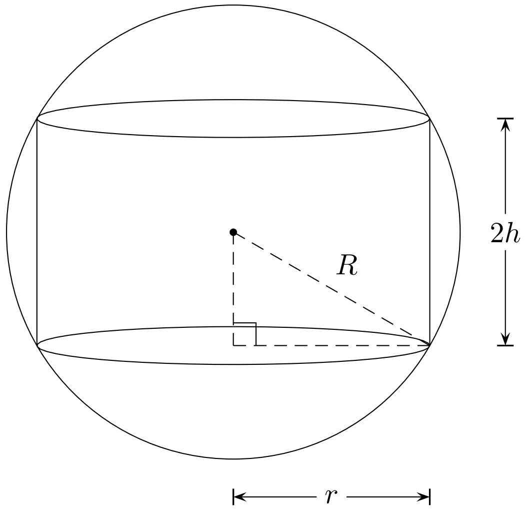
- As $x \to \pm\infty$, $y \to 0$
- The vertical asymptotes of its graph are $x = -2$ and $x = 2$
- The table below shows the first and second derivatives at various points:

| $x$ | $x < -2$ | $-2 < x < 0$ | $x = 0$ | $0 < x < 2$ | $x > 2$ |
|---|---|---|---|---|---|
| $f(x)$ | | | 1 | | |
| $f'(x)$ | $< 0$ | $< 0$ | 0 | $> 0$ | $> 0$ |
| $f''(x)$ | $< 0$ | $> 0$ | $> 0$ | $> 0$ | $< 0$ |

Sketch $y = f(x)$, using the properties in the table above.

**Question 17** (7 marks)

The diagram below shows a cylinder of height $2h$ and radius $r$ inscribed within a sphere of fixed radius $R$.




Let the volume of the cylinder be $V$.

(a) Show that $V = 2\pi(R^2h - h^3)$. **2**

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(b) Show that $V$ is maximised when the height of the cylinder is $\dfrac{2R}{\sqrt{3}}$. **3**

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**Examination continues overleaf**...

(c) Find the ratio of the radius of the cylinder to the radius of the sphere when the volume is maximised. **2**

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**Question 18** (6 marks)

Differentiate with respect to $x$:

(a) $\dfrac{3x^2+1}{x+4}$ **2**

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(b) $x\sin^2 x$ **2**

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(c) $\ln \sqrt{4x^2 - 1}$ **2**

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**Question 19** (3 marks)

Let $f(x) = e^{-kx} + 3x$ where $k$ is a positive rational number.

(a) Find, in terms of $k$, the $x$-coordinate of the stationary point of the graph of $y = f(x)$. **2**

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(b) State the values of $k$ such that the $x$-coordinate of this stationary point is a positive value. **1**

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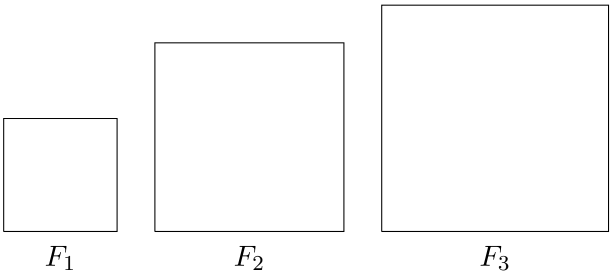
**Examination continues overleaf**. . .

**Question 20** (3 marks)

In the diagram below, $F_1$, $F_2$, $F_3$ are square frames. The perimeter of the first frame is 16 cm. The perimeter of each square frame afterwards is 4 cm longer than the perimeter of the previous frame.




(a) Find the perimeter of $F_{12}$. **1**

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(b) A thin metal wire of length 2000 cm is cut into pieces and these pieces are then bent to form the above square frames. **2**

Find the greatest number of distinct square frames that can be formed.

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**Question 21** (4 marks)

$3, k, \frac{3}{2}$ are the first three terms of a geometric sequence, where $k$ is a positive number.

(a) Show that $k = \frac{3}{\sqrt{2}}$ **1**

(b) Find the 7th term of this sequence. **1**

(c) Show that the limiting sum is equal to **2**

$$6 + \sqrt{18}$$

**Examination continues overleaf...**

**Question 22** (5 marks)

(a) By sketching the graph of $y = \sqrt{4 - x^2}$, show that **3**

$$\int_0^1 \sqrt{4 - x^2}\, dx = \frac{\sqrt{3}}{2} + \frac{\pi}{3}$$

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(b) Hence or otherwise, find $\displaystyle\int_2^3 \sqrt{16x - 4x^2}\, dx$. **2**

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**Examination continues overleaf...**

**Question 23** (8 marks)

The derivative of a function $y = f(x)$ is given by $f'(x) = 3x^2 - 2x - 1$.

(a) Find the $x$-values of the two stationary points of $y = f(x)$, and determine the nature of the stationary points. **3**

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(b) The curve passes through the point $(0, 3)$. Find an expression for $f(x)$. **2**

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(c) For what values of $x$ is the curve concave up? **1**

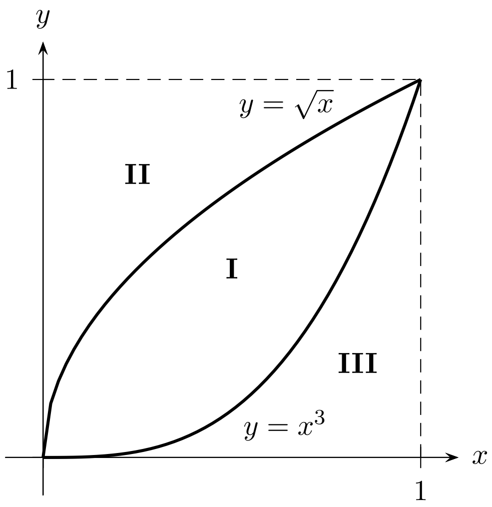
(d) Draw a sketch of the curve $y = f(x)$, clearly indicating the coordinates of the stationary points. **2**

(You do **NOT** need to find the $x$-intercepts)

**Examination continues overleaf...**

**Question 24** (3 marks)

The diagram below shows a unit square target for shooting on the Cartesian Plane. The target is divided into three regions. I, II and III by the curves $y = \sqrt{x}$ and $y = x^3$. **3**




Find the areas of all three regions.

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**Question 25** (3 marks)

Seb's company produces motors for refrigerators. There are two assembly lines, Line A and Line B. 5% of the motors assembled on Line A are faulty and 8% of the motors assembled on Line B are faulty.

In one hour, 40 motors are produced from Line A and 50 motors are produced from Line B. At the end of an hour, one motor is selected at random from all the motors that have been produced during that hour.

(a) What is the probability that the selected motor is faulty? **2**

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(b) The selected motor is faulty. **1**
What is the probability that it was assembled on Line A?

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**Examination continues overleaf**...

**Question 26** (5 marks)

The table below shows the probability distribution of a discrete random variable $X$.

| $x$ | 0 | 2 | 4 | 5 | 8 | 9 |
|---|---|---|---|---|---|---|
| $P(X = x)$ | $k^2$ | 0.16 | 0.18 | 0.3 | $k$ | 0.12 |

(a) Show that $k^2 + k - 0.24 = 0$. **1**

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(b) Hence, show and briefly explain why $k = 0.2$. **2**

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(c) Calculate the expected value. **2**

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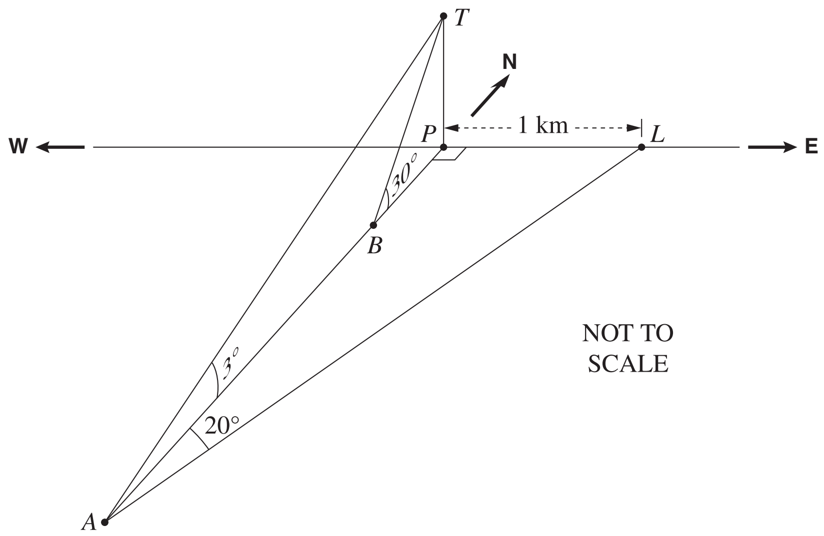
**Question 27** (5 marks)

A boat is sailing due north from a point $A$ towards a point $P$ on the shore line.
The shore line runs from west to east.
In the diagram, $T$ represents a tree on a cliff vertically above $P$, and $L$ represents a landmark on the shore. The distance $PL$ is 1 km.

From $A$ the point $L$ is on a bearing of 020°, and the angle of elevation to $T$ is 3°.

After sailing for some time the boat reaches a point $B$, from which the angle of elevation to $T$ is 30°.




(a) Show that $BP = \dfrac{\sqrt{3}\tan 3^\circ}{\tan 20^\circ}$. **3**

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(b) Find the distance $AB$. Give your answer to 2 significant figures. **2**

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**Examination continues overleaf...**

**Question 28** (6 marks)

The price $P(t)$ in cents per litre of unleaded petrol during an average year in Broome WA, can be modelled by the function

$$P(t) = 180 + 44\sin\left(\frac{2\pi t}{183}\right)$$

where $t$ is the number of days after 1 July 2023, for $0 \le t \le 366$.

(a) What is the maximum price of petrol during the year? **1**

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(b) Sketch the function $P(t)$ for $0 \le t \le 366$. **2**

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**Examination continues overleaf...**

(c) What are the values of $t$ for when petrol will cost 202 cents per litre. **3**

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**Examination continues overleaf**…

**Question 29** (4 marks)

(a) Prove that $(1 - \sin x)(\sec x + \tan x) = \cos x$. **2**

(b) Hence, or otherwise, evaluate $\displaystyle\int_0^{\frac{\pi}{2}} \sin^2 x(1 - \sin x)(\sec x + \tan x)\, dx$. **2**

**Question 30** (7 marks)

The population size $P$ of a species of birds living in a wildlife preserve increases at a rate of

$$\frac{dP}{dt} = 9e^{\frac{t^2}{5}} - 2t \quad \text{for } t \geq 0$$

where $t$ is the time in months. It is known that the initial population of the bird is 34.

(a) Use the trapezoidal rule with 4 subintervals to estimate $\int_0^4 e^{\frac{t^2}{5}}\, dt$. **3**

Hence, show that at $t = 4$, there are approximately 218 birds.

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After 4 months, a coal mine was built near the wildlife preserve and pollution from the mine affects the population of the birds from $t = 4$ onwards.

Environmental modelling now reveals the population $P$ can now be modelled by the equation

$$P = Ate^{-0.05t} - 100 \qquad t \geq 4$$

(b) Using part (a), show that $A \approx 97$. **1**

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**Examination continues overleaf...**

(c) Determine the maximum population size after the mine has been built. Leave your answer correct to the nearest integer. **3**

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**End of paper.**

NORMANHURST BOYS HIGH SCHOOL

# MATHEMATICS ADVANCED

2023 Year 12 Course Assessment Task 4 (Trial HSC Examination)
Wednesday, 16 August 2023

**General instructions**

- Working time – 3 hours.
  (plus 10 minutes reading time)
- Write using blue or black pen. Where diagrams are to be sketched, these may be done in pencil.
- NESA approved calculators may be used.
- Attempt **all** questions.

**SECTION I**

- Mark your answers on the answer grid provided (on page 29)

**SECTION II**

- All necessary working should be shown in every question. Marks may be deducted for illegible or incomplete working.

**NESA STUDENT #:** Sample Solutions.

**Class** (please ✔)

◯ 12MAA.1 – Mr Lam
◯ 12MAX.2 – Ms Lee
◯ 12MAX.1 – Ms C. Kim
◯ 12MAX.3 – Ms J. Kim

Marker's use only.

| QUESTION | 1-10 | 11-14 | 15-17 | 18-19 | 20-21 | 22-24 | 25-26 | 27-28 | 29-30 | Total |
|---|---|---|---|---|---|---|---|---|---|---|
| MARKS | 10 | 15 | 13 | 9 | 7 | 16 | 8 | 11 | 11 | 100 |

# Section I

**10 marks**
**Attempt Question 1 to 10**
**Allow approximately 15 minutes for this section**

Mark your answers on the answer grid provided (labelled as page 29).

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**Questions** **Marks**

**1.** Simplify $\sqrt{x^2+2x+1}-\sqrt{x^2-2x+1}$ **1**

(A) $-2$

(B) $2$

(C) $2x$

(D) $2x+2$

**2.** Using the table below, which of the following is an expression for $E\left(X^2\right)$? **1**

| $x$ | 10 | 20 | 30 | 40 | 50 |
|---|---|---|---|---|---|
| $P(X=x)$ | $1-3a$ | $a$ | $1-9a$ | $1-10a$ | $a$ |

(A) $10^2(1-3a)+20^2a+30^2(1-9a)+40^2(1-10a)+50^2a$

(B) $10(1-3a)^2+20a^2+30(1-9a)^2+40(1-10a)^2+50a^2$

(C) $10^2(1-3a)^2+20^2a^2+30^2(1-9a)^2+40^2(1-10a)^2+50^2a^2$

(D) $\left[10(1-3a)+20a+30(1-9a)+40(1-10a)+50a\right]^2$

**3.** Which of the following represents the solutions for the equation $2^{2x}-5(2^x)+4=0$? **1**

(A) $x=0$ or $x=1$

(B) $x=0$ or $x=2$

(C) $x=\log_2 1$ or $x=\log_2 2$

(D) $x=1$ or $x=4$

**4.** What is the period of the function $f(x)=-3\sin\left(\dfrac{\pi x}{5}\right)$? **1**

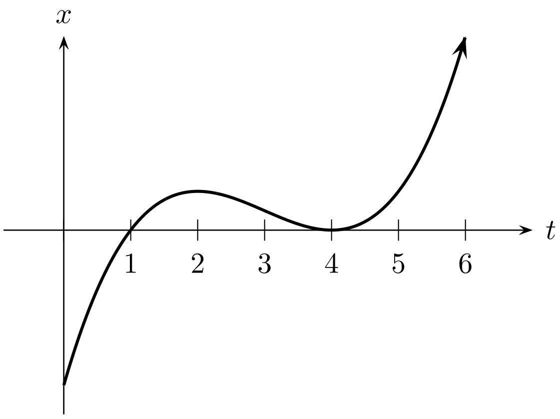
(A) 5

(B) $5\pi$

(C) 10

(D) $10\pi$

**5.** The displacement, $x$ metres, from the origin of a particle moving in a straight line at any time, $t$ seconds, is shown in the graph below. **1**




When was the particle at rest?

(A) $t = 0$

(B) $t = 1$ and $t = 4$

(C) $t = 2$ and $t = 4$

(D) $t = 1$, $t = 2$ and $t = 4$

**6.** Evaluate $\lim_{x \to 3} \dfrac{x^2 - 2x - 3}{x - 3}$ **1**

(A) 0

(B) undefined

(C) 4

(D) 1

**7.** Which of the following represents the domain and range of the function **1**

$$f(x) = \sqrt{4 - x^2}$$

(A) Domain: $-2 \le x \le 2$, Range: $0 \le y \le 2$

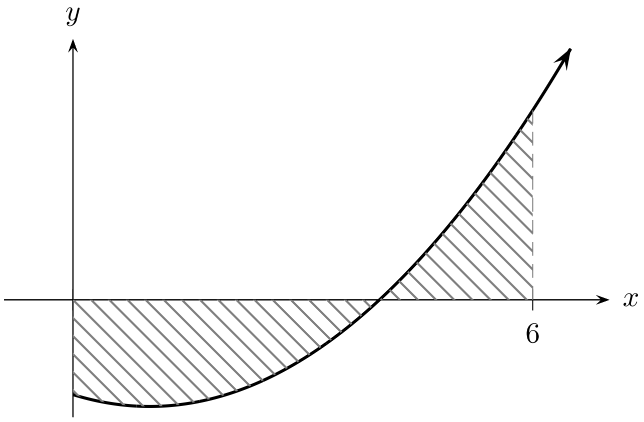
(B) Domain: $-2 \le x \le 2$, Range: $-2 \le y \le 2$

(C) Domain: $0 \le x \le 2$, Range: $-4 \le y \le 4$

(D) Domain: $0 \le x \le 2$, Range: $0 \le y \le 4$

**Examination continues overleaf...**

**8.** The diagram below shows the graph of $y = x^2 - 2x - 8$. **1**




What is the correct expression for the area bounded by the $x$-axis and the curve $y = x^2 - 2x - 8$ between $0 \leq x \leq 6$?

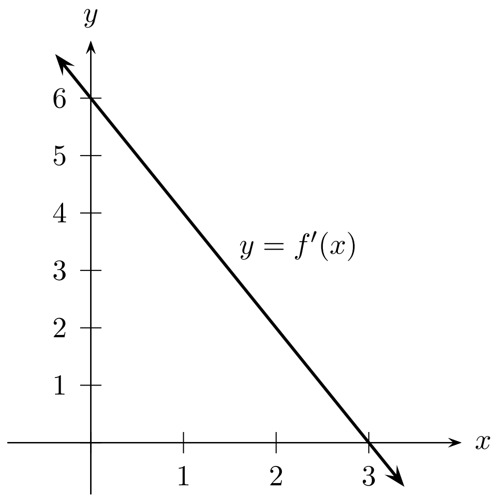
(A) $A = \int_0^5 \left(x^2 - 2x - 8\right)\, dx + \left| \int_5^6 \left(x^2 - 2x - 8\right)\, dx \right|$

(B) $A = \int_0^4 \left(x^2 - 2x - 8\right)\, dx + \left| \int_4^6 \left(x^2 - 2x - 8\right)\, dx \right|$

(C) $A = \left| \int_0^5 \left(x^2 - 2x - 8\right)\, dx \right| + \int_5^6 \left(x^2 - 2x - 8\right)\, dx$

(D) $A = \left| \int_0^4 \left(x^2 - 2x - 8\right)\, dx \right| + \int_4^6 \left(x^2 - 2x - 8\right)\, dx$

**9.** The graph of $y = f'(x)$ is shown below. **1**




The curve $y = f(x)$ has a maximum value of 20. Which of the following is the equation of $f(x)$?

(A) $y = x^2 - 6x + 11$

(B) $y = x^2 - 6x + 20$

(C) $y = -x^2 + 6x + 20$

(D) $y = -x^2 + 6x + 11$

**10.** Which of the following is equal to $\log_c(a) + \log_a(b) + \log_b(c)$? **1**

(A) $\dfrac{1}{\log_c(a)} + \dfrac{1}{\log_a(b)} + \dfrac{1}{\log_b(c)}$

(B) $\dfrac{1}{\log_a(c)} + \dfrac{1}{\log_b(a)} + \dfrac{1}{\log_c(b)}$

(C) $-\dfrac{1}{\log_a(b)} - \dfrac{1}{\log_b(c)} - \dfrac{1}{\log_c(a)}$

(D) $\dfrac{1}{\log_a(a)} + \dfrac{1}{\log_b(b)} + \dfrac{1}{\log_c(c)}$

**Examination continues overleaf...**

## Section II

**90 marks**
**Attempt Question 11 to 30**
**Allow approximately 2 hours and 45 minutes for this section**

Write your answers in the space provided.

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**Question 11** (5 marks)

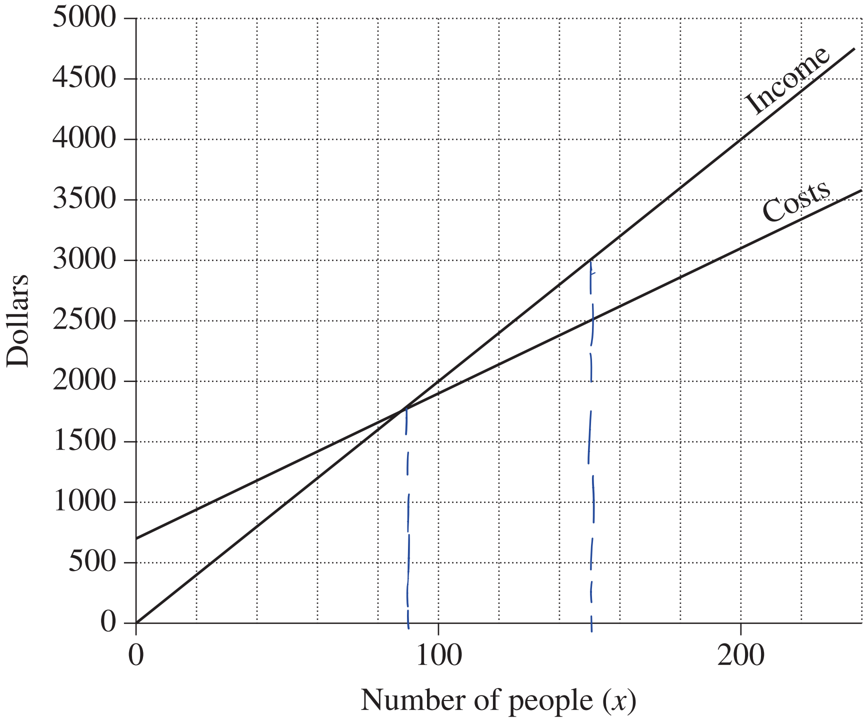
Sam and Jammy are planning a fund-raising dance. They can hire a hall for \$400 and a band for \$300. Refreshments will cost them \$12 per person.

(a) Write a formula for the cost (\$$C$) of running the dance for $x$ people. **1**

$C = 700 + 12x$

The graph shows planned income and costs when the ticket price is \$20.

**Planned income and costs for the dance**




(b) Estimate the minimum number of people needed at the dance to cover the costs. **1**

$\approx 90$

(c) How much profit will be made if 150 people attend the dance? **1**

profit $= 150 \times 20 - (700 + 12 \times 150)$

\$500 (from graph) OR $= 500$.

(d) Sam and Jammy plan to sell 200 tickets. They want to make a profit of \$1500. What should be the price of a ticket, assuming all 200 tickets will be sold? **2**

let $t$ = ticket price

$1500 = 200t - (700 + 12 \times 200)$ (1)

$200t = 4600$

$\therefore t = 23$

$\therefore$ \$23 / ticket (1)

**Question 12** (4 marks)

The graph of the function $f(x)$ is obtained from the graph of the function **4**

$$g(x) = 3\cos\left(x - \frac{\pi}{6}\right)$$

by applying the following transformations.

1. A dilation of a factor of $\frac{1}{2}$ from the $x$-axis
2. A reflection in the $y$-axis
3. A translation of $\frac{\pi}{6}$ units in the negative $x$ direction
4. A translation of 4 units in the negative $y$ direction

Find the rule of $f(x)$.

1. $h(x) = \frac{g(x)}{\frac{1}{2}} = \frac{1}{2} \times 3\cos\left(x - \frac{\pi}{6}\right)$

$= \frac{3}{2}\cos\left(x - \frac{\pi}{6}\right)$ (1)

2. $j(x) = h(-x) = \frac{3}{2}\cos\left(-x - \frac{\pi}{6}\right)$ (1)

3. $k(x) = j\left(x + \frac{\pi}{6}\right) = \frac{3}{2}\cos\left(-\left(x + \frac{\pi}{6}\right) - \frac{\pi}{6}\right)$

$= \frac{3}{2}\cos\left(-x - \frac{\pi}{3}\right)$ (1)

4. $f(x) = k(x) - 4 = \frac{3}{2}\cos\left(-x - \frac{\pi}{3}\right) - 4$ (1)

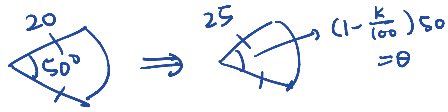
$\therefore f(x) = \frac{3}{2}\cos\left(-x - \frac{\pi}{3}\right) - 4$

**Examination continues overleaf...**

**Question 13** (3 marks)

A sector has radius length of 20 cm and the angle subtended at the centre is 50°. The radius of this sector increased by 25% and its angle at the centre is decreased by $k$%. **3**

If the area of the sector remains unchanged find the value of $k$.




$\pi \times 20^2 \times \frac{50}{360} = \pi \times 25^2 \times \frac{\theta}{360}$ (1)

$\therefore \theta = 32$

$50\left(1 - \frac{k}{100}\right) = 32$ (1)

$1 - \frac{k}{100} = \frac{32}{50}$

$\frac{k}{100} = 0.36$

$\therefore k = 36$ (1)

**Question 14** (3 marks)

Solve for $x$: **3**

$$2\ln(x+2) - \ln x = \ln(2x+1) \quad \text{where} x > 0$$

$\ln(x+2)^2 - \ln x = \ln(2x+1)$

$\ln \frac{(x+2)^2}{x} = \ln(2x+1)$ (1)

$\frac{x^2+4x+4}{x} = 2x+1$

$x^2+4x+4 = 2x^2+x$ (1)

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$x = 4, -1$. but $x > 0$ (1)

$\therefore x = 4$ only.

**Question 15** (3 marks)

Find the equation of the normal to the curve $y = 2(5x-4)^4$ at $x = 1$. **3**
Express your answer in general form $ax + by + c = 0$.

$\frac{dy}{dx} = 2 \times 4(5x-4)^3 \times 5$

$= 40(5x-4)^3$ ①

at $x = 1$, $m = 40$

$\therefore m_\perp = -\frac{1}{40}$ ①

at $x = 1$, $y = 2$

$\therefore y - 2 = -\frac{1}{40}(x-1)$

$-40y + 80 = x - 1$

$\therefore x + 40y - 81 = 0$. ①

**Examination continues overleaf…**

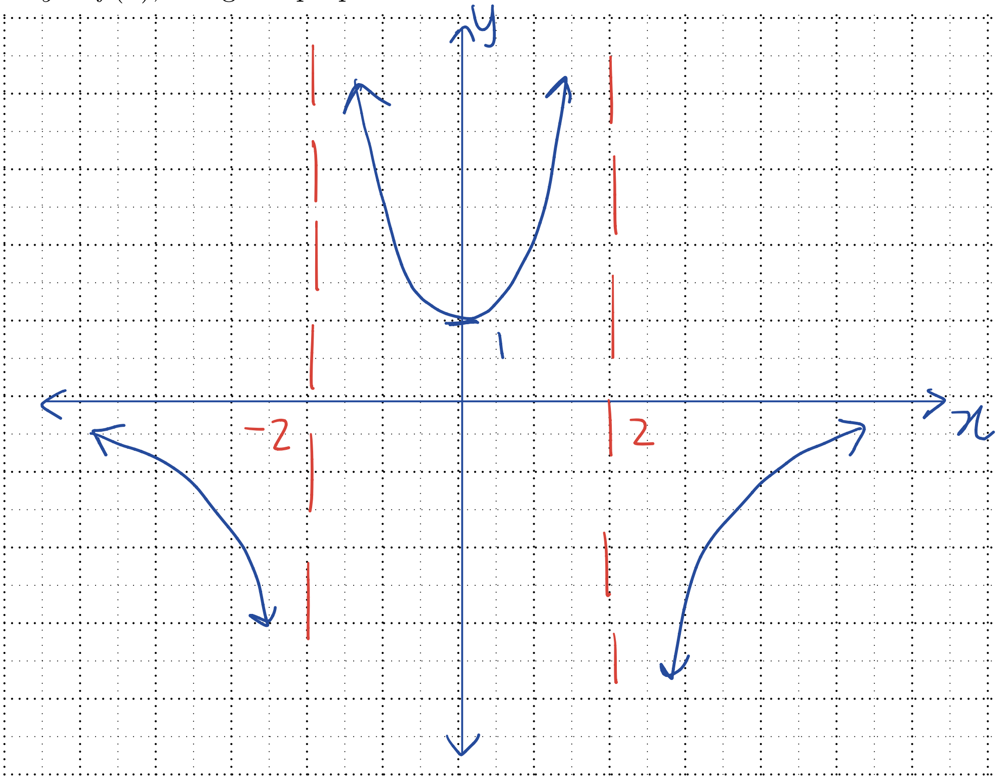
**Question 16** (3 marks)

A rational function $f(x)$ has the following properties: **3**

- As $x \to \pm\infty$, $y \to 0$
- The vertical asymptotes of its graph are $x = -2$ and $x = 2$
- The table below shows the first and second derivatives at various points:

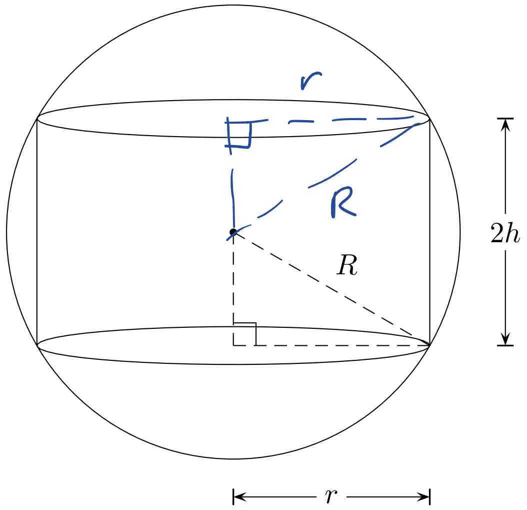
| $x$ | $x < -2$ | $-2 < x < 0$ | $x = 0$ | $0 < x < 2$ | $x > 2$ |
|---|---|---|---|---|---|
| $f(x)$ | | | 1 | | |
| $f'(x)$ | $< 0$ | $< 0$ | 0 | $> 0$ | $> 0$ |
| $f''(x)$ | $< 0$ | $> 0$ | $> 0$ | $> 0$ | $< 0$ |

Sketch $y = f(x)$, using the properties in the table above.



**Question 17** (7 marks)

The diagram below shows a cylinder of height $2h$ and radius $r$ inscribed within a sphere of fixed radius $R$.




Let the volume of the cylinder be $V$.

(a) Show that $V = 2\pi(R^2h - h^3)$. **2**

$V = \pi r^2 h$

$h_{cylinder} = 2h$, By Pythagoras' Thm.

$r^2 = R^2 - h^2$ (by congruent Δs) ①

$\therefore V = \pi(R^2 - h^2) \times 2h$

$= 2\pi(R^2h - h^3)$ ①

(b) Show that $V$ is maximised when the height of the cylinder is $\dfrac{2R}{\sqrt{3}}$. **3**

$\dfrac{dV}{dh} = 2\pi(R^2 - 3h^2)$

when $\dfrac{dV}{dh} = 0$; $2\pi(R^2 - 3h^2) = 0$

$R^2 = 3h^2$ ①

$h^2 = \dfrac{R^2}{3}$

$\therefore h = \dfrac{R}{\sqrt{3}}$ $(R > 0)$ ①

$\therefore$ height of cylinder $= 2h = \dfrac{2R}{\sqrt{3}}$ ①

**Examination continues overleaf...**

(c) Find the ratio of the radius of the cylinder to the radius of the sphere when the volume is maximised. **2**

$r^2 = R^2 - h^2$

$= R^2 - \frac{R^2}{3}$ $\quad (h^2 = \frac{R^2}{3}$ from b) $\quad$ ①

$= \frac{2R^2}{3}$

$\therefore r : R = \frac{\sqrt{2}}{\sqrt{3}}R : R$

$= \sqrt{2} : \sqrt{3}$ ①

**Question 18** (6 marks)

Differentiate with respect to $x$:

(a) $\dfrac{3x^2+1}{x+4}$ **2**

$u = 3x^2 + 1 \qquad v = x+1$

$u' = 6x \qquad v = 1$

$\frac{d}{dx}\left(\frac{3x^2+1}{x+4}\right) = \frac{6x(x+1) - (3x^2+1)}{(x+1)^2}$ ①

$= \frac{6x^2+6x-3x^2-1}{(x+1)^2}$

$= \frac{3x^2+6x-1}{(x+1)^2}$ ①

(b) $x\sin^2 x$ **2**

$u = x \qquad v = \sin^2 x$

$u' = 1 \qquad v' = 2\sin x\cos x$

$\frac{d}{dx}(x\sin^2 x) = \sin^2 x + 2x\sin x\cos x$

① ①

(c) $\ln\sqrt{4x^2-1}$ **2**

$= \ln(4x^2-1)^{\frac{1}{2}}$

$= \frac{1}{2}\ln(4x^2-1)$

$\frac{d}{dx}\left(\frac{1}{2}\ln(4x^2-1)\right) = \frac{1}{2} \times \frac{8x}{4x^2-1}$ (1)

$= \frac{4x}{4x^2-1}$ (1)

**Question 19** (3 marks)

Let $f(x) = e^{-kx} + 3x$ where $k$ is a positive rational number.

(a) Find, in terms of $k$, the $x$-coordinate of the stationary point of the graph of $y = f(x)$. **2**

$f'(x) = -ke^{-kx} + 3$ (1)

when $f'(x) = 0$;

$-ke^{-kx} + 3 = 0$

$ke^{-kx} = 3$

$e^{-kx} = \frac{3}{k}$

$-kx = \ln\left|\frac{3}{k}\right|$

$x = -\frac{1}{k}\ln\left(\frac{3}{k}\right)$ $(k>0)$. (1) OR $= \frac{1}{k}\ln\left(\frac{k}{3}\right)$.

(b) State the values of $k$ such that the $x$-coordinate of this stationary point is a positive value. **1**

$\frac{1}{k}\ln\left(\frac{k}{3}\right) > 0$

$\ln\left(\frac{k}{3}\right) > 0$

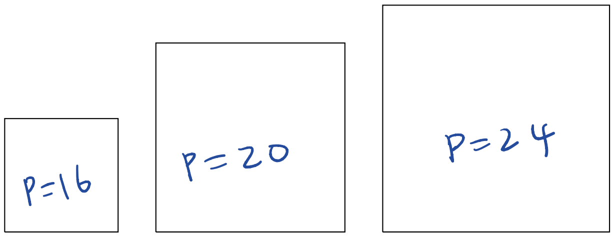
$\frac{k}{3} > 1$

$\therefore k > 3$.

**Examination continues overleaf...**

**Question 20** (3 marks)

In the diagram below, $F_1$, $F_2$, $F_3$ are square frames. The perimeter of the first frame is 16 cm. The perimeter of each square frame afterwards is 4 cm longer than the perimeter of the previous frame.




$F_1$ $F_2$ $F_3$

(a) Find the perimeter of $F_{12}$. **1**

16, 20, 24, ...

$a=16$, $d=4$

$T_{12} = 16 + (12-1) \times 4$

$= 60$

$\therefore P = 60$ cm

(b) A thin metal wire of length 2000 cm is cut into pieces and these pieces are then bent to form the above square frames. **2**

Find the greatest number of distinct square frames that can be formed.

$S_n < 2000$ $n = ?$

$\frac{n}{2}[2\times16 + (n-1)\times4] < 2000$

$32n + 4n^2 - 4n < 4000$

$4n^2 + 28n - 4000 < 0$

$n^2 + 7n - 1000 < 0$ (1)

$\therefore n = \frac{-7 \pm \sqrt{7^2 + 4\times1000}}{2}$

$= 28.315, -35.3158...$ (but $n > 0$)

-35.3158 28.315

$-35.3158 < n < 28.315$

$\therefore n = 28$ (1)

$\therefore$ 28 full boxes.

**Question 21** (4 marks)

$3, k, \frac{3}{2}$ are the first three terms of a geometric sequence, where $k$ is a positive number.

(a) Show that $k = \frac{3}{\sqrt{2}}$ **1**

$$\frac{k}{3} = \frac{\frac{3}{2}}{k}$$

$$k^2 = \frac{9}{2}$$

$$k = \frac{3}{\sqrt{2}} \quad (k>0).$$

(b) Find the 7th term of this sequence. **1**

$$T_7 = 3\left(\frac{1}{\sqrt{2}}\right)^6$$

$$= \frac{3}{8}$$

(c) Show that the limiting sum is equal to **2**

$$6 + \sqrt{18}$$

$$S_\infty = \frac{a}{1-r}$$

$$= \frac{3}{1-\frac{1}{\sqrt{2}}} \quad (1)$$

$$= \frac{3}{\frac{\sqrt{2}-1}{\sqrt{2}}}$$

$$= \frac{3\sqrt{2}}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1}$$

$$= \frac{3\times 2+3\sqrt{2}}{2-1} = 6+3\sqrt{2}$$

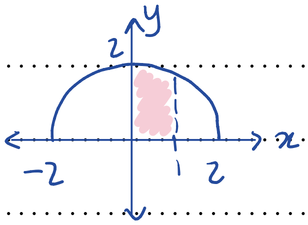
$$= 6+\sqrt{18} \quad (1)$$

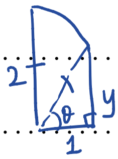
**Examination continues overleaf...**

**Question 22** (5 marks)

(a) By sketching the graph of $y = \sqrt{4 - x^2}$, show that **3**

$$\int_0^1 \sqrt{4 - x^2}\,dx = \frac{\sqrt{3}}{4} + \frac{\pi}{12}$$


$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}$

$\sin\frac{\pi}{3} = \frac{y}{2}$

$\therefore y = 2\sin\frac{\pi}{3}$

$= \sqrt{3}$

$\therefore A_\Delta = \frac{1}{2} \times 1 \times \sqrt{3}$ (1)

$= \frac{\sqrt{3}}{2}$

$A_{sector} = \frac{1}{2} \times 2^2 \times \frac{\pi}{6}$ (1)

$= \frac{\pi}{3}$

$\therefore \int_0^1 \sqrt{4-x^2}\,dx = \frac{\sqrt{3}}{2} + \frac{\pi}{3}$ (1)

(b) Hence or otherwise, find $\int_2^4 \sqrt{16x - 4x^2}\,dx$. **2**


$\int_2^4 \sqrt{4(4x-x^2)}\,dx = 2\int_2^4 \sqrt{4x-x^2}$

$f(x) = \sqrt{4-x^2}$ (1)

$f(x-2) = \sqrt{4-(x-2)^2}$

$= \sqrt{4-(x^2-4x+4)}$

$= \sqrt{4x-x^2}$

$\int_2^3 \sqrt{16-4x^2}\,dx = 2\times\left(\frac{\sqrt{3}}{2} + \frac{\pi}{3}\right) = \sqrt{3} + \frac{2\pi}{3}$ (1)

**Examination continues overleaf...**

**Question 23** (8 marks)

The derivative of a function $y = f(x)$ is given by $f'(x) = 3x^2 - 2x - 1$.

(a) Find the $x$-values of the two stationary points of $y = f(x)$, and determine the nature of the stationary points. **3**

when $f'(x) = 0$, $3x^2 - 2x - 1 = 0$

$(3x+1)(x-1) = 0$

$\therefore x = -\frac{1}{3}, 1$ (1)

$f''(x) = 6x - 2$

$f''(-\frac{1}{3}) = -4 < 0$ $\therefore$ max turning point at $x = -\frac{1}{3}$ (1)

$f''(1) = 4 > 0$ $\therefore$ min turning point at $x = 1$ (1)

(b) The curve passes through the point $(0, 3)$. Find an expression for $f(x)$. **2**

$f'(x) = 3x^2 - 2x - 1$

$f(x) = \frac{3x^3}{3} - \frac{2x^2}{2} - x + C$

$= x^3 - x^2 - x + C$ (1)

at $(0,3)$; $3 = 0^3 - 0^2 - 0 + C$

$\therefore C = 3$

$\therefore f(x) = x^3 - x^2 - x + 3$ (1)

(c) For what values of $x$ is the curve concave up? **1**

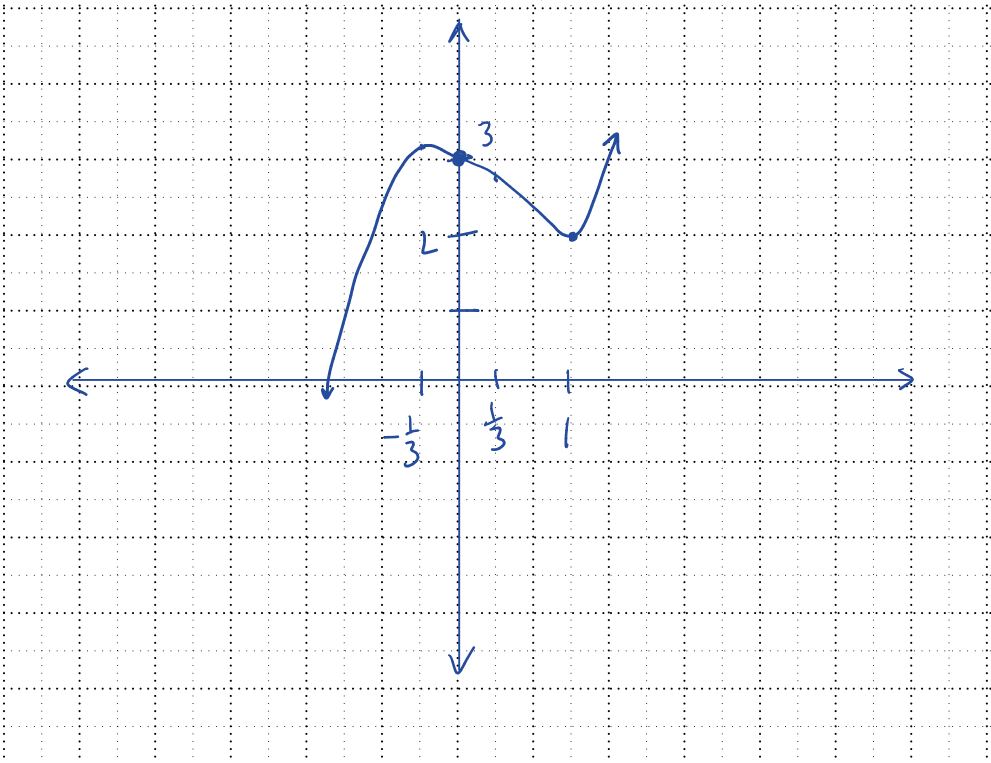
when $f''(x) = 0$, $6x - 2 = 0$

$x = \frac{1}{3}$

$\therefore x > \frac{1}{3}$ ($x = -\frac{1}{3}$ max, $x = 1$ min).

(d) Draw a sketch of the curve $y = f(x)$, clearly indicating the coordinates of the stationary points. **2**

(You do **NOT** need to find the $x$-intercepts)




at $x = -\frac{1}{3}$, $y = \frac{86}{27}$

$\approx 3.185$ (max).

$x = 1$, $y = 2$ (min).

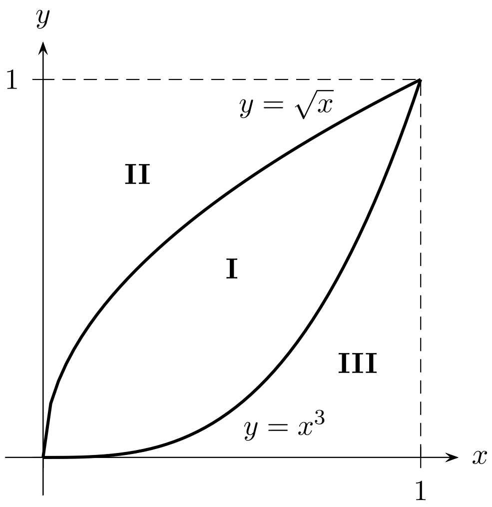
**Examination continues overleaf...**

$(0,3)$ & POI at $x = \frac{1}{3}$, $y = \frac{70}{27}$

$\approx 2.59$

**Question 24** (3 marks)

The diagram below shows a unit square target for shooting on the Cartesian Plane. The target is divided into three regions. I, II and III by the curves $y = \sqrt{x}$ and $y = x^3$. **3**




Find the areas of all three regions.

Region III $= \int_0^1 x^3 \, dx$

$= \left[\frac{x^4}{4}\right]_0^1$

$= \frac{1}{4}$ (1)

Region I $= \int_0^1 x^{\frac{1}{2}} \, dx - \frac{1}{4}$

$= \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1 - \frac{1}{4}$

$= \frac{2}{3} - \frac{1}{4}$

$= \frac{5}{12}$ (1)

Region II $= 1 - \frac{1}{4} - \frac{5}{12}$

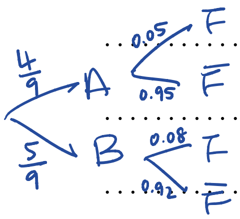
$= \frac{1}{3}$ (1)

**Question 25** (3 marks)

Seb's company produces motors for refrigerators. There are two assembly lines, Line A and Line B. 5% of the motors assembled on Line A are faulty and 8% of the motors assembled on Line B are faulty.

In one hour, 40 motors are produced from Line A and 50 motors are produced from Line B. At the end of an hour, one motor is selected at random from all the motors that have been produced during that hour.

(a) What is the probability that the selected motor is faulty? **2**




$P(\text{Faulty}) = \frac{4}{9} \times \frac{5}{100} + \frac{5}{9} \times \frac{8}{100}$ (1)

$= \frac{1}{15}$. (1)

(b) The selected motor is faulty.
What is the probability that it was assembled on Line A? **1**

$P(\text{Line A} \mid \text{Faulty}) = \frac{P(A \cap F)}{P(F)}$

$= \frac{\frac{4}{9} \times \frac{5}{100}}{\frac{1}{15}}$

$= \frac{1}{3}$. (1)

**Examination continues overleaf...**

**Question 26** (5 marks)

The table below shows the probability distribution of a discrete random variable $X$.

| $x$ | 0 | 2 | 4 | 5 | 8 | 9 |
|---|---|---|---|---|---|---|
| $P(X = x)$ | $k^2$ | 0.16 | 0.18 | 0.3 | $k$ | 0.12 |

(a) Show that $k^2 + k - 0.24 = 0$. **1**

$k^2 + 0.16 + 0.18 + 0.3 + k + 0.12 = 1$

$k^2 + k - 0.24 = 0.$

(b) Hence, show and briefly explain why $k = 0.2$. **2**

$k = \frac{-1 \pm \sqrt{1 + 4 \times 0.24}}{2}$

$= 0.2$ or $-1.2$ ①

but $k \geq 0$

$\therefore k = 0.2$ only ①

(c) Calculate the expected value. **2**

$E(X) = 0 \times 0.2^2 + 2 \times 0.16 + 4 \times 0.18 + 5 \times 0.3 + 8 \times 0.2 + 9 \times 0.12$ ①

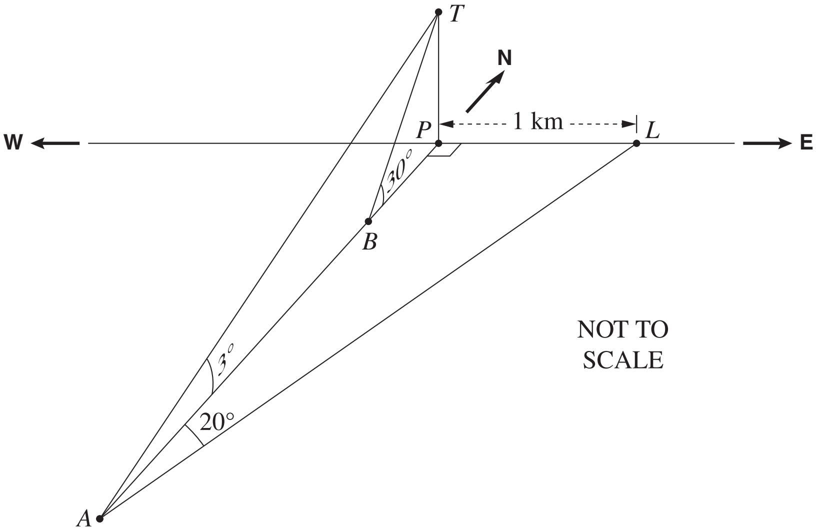
$= 5.22$ ①

**Question 27** (5 marks)

A boat is sailing due north from a point $A$ towards a point $P$ on the shore line.
The shore line runs from west to east.
In the diagram, $T$ represents a tree on a cliff vertically above $P$, and $L$ represents a landmark on the shore. The distance $PL$ is 1 km.

From $A$ the point $L$ is on a bearing of 020°, and the angle of elevation to $T$ is 3°.

After sailing for some time the boat reaches a point $B$, from which the angle of elevation to $T$ is 30°.




(a) Show that $BP = \dfrac{\sqrt{3}\tan 3^\circ}{\tan 20^\circ}$. **3**

$\tan 30^\circ = \dfrac{TP}{BP}$ $\quad\quad$ $\tan 3^\circ = \dfrac{TP}{AP}$ (1)

$TP = BP\tan 30^\circ$ $\quad\quad$ $TP = AP\tan 3^\circ$

$\therefore BP\tan 30^\circ = AP\tan 3^\circ$

$BP = \dfrac{AP\tan 3^\circ}{\tan 30^\circ}$ $\quad\quad$ $\tan 20^\circ = \dfrac{1}{AP}$, $AP = \dfrac{1}{\tan 20^\circ}$ (1)

$= \dfrac{\frac{\tan 3^\circ}{\tan 20^\circ}}{\tan 30^\circ} = \dfrac{\sqrt{3}\tan 3^\circ}{\tan 20^\circ}$ (1)

(b) Find the distance $AB$. Give your answer to 2 significant figures. **2**

$AB = AP - BP$

$= \dfrac{1}{\tan 20^\circ} - \dfrac{\sqrt{3}\tan 3^\circ}{\tan 20^\circ}$ (1)

$= 2.4980\ldots$

$= 2.5$ (2sf). (1)

**Examination continues overleaf...**

**Question 28** (6 marks)

The price $P(t)$ in cents per litre of unleaded petrol during an average year in Broome WA, can be modelled by the function

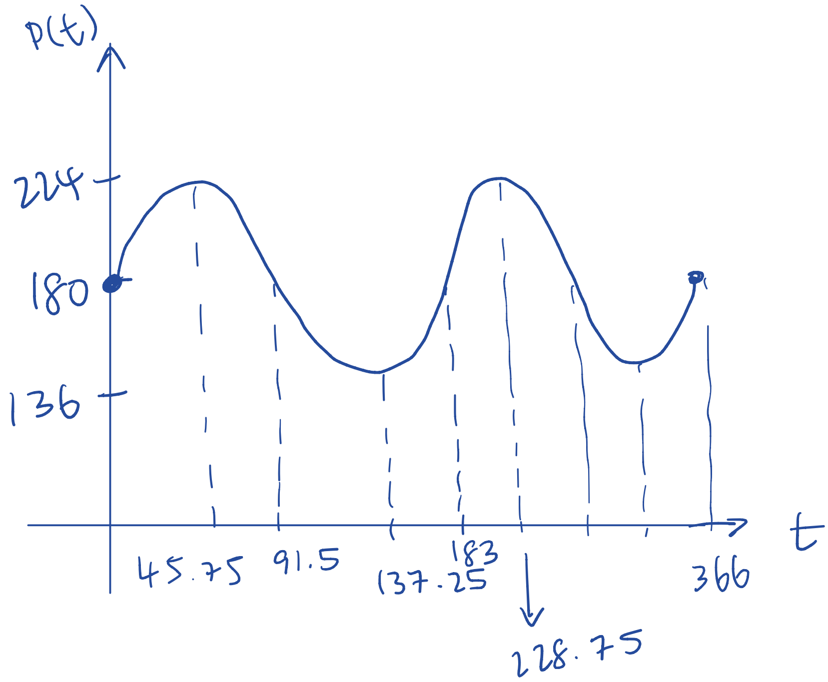
$$P(t) = 180 + 44\sin\left(\frac{2\pi t}{183}\right)$$

where $t$ is the number of days after 1 July 2023, for $0 \leq t \leq 366$.

(a) What is the maximum price of petrol during the year? **1**

$180 + 44 = 224$.

(b) Sketch the function $P(t)$ for $0 \leq t \leq 366$. **2**




$T = \dfrac{2\pi}{\frac{2\pi}{183}} = 183$

$\therefore$ 45.75, 91.5, 137.25, 183.

**Examination continues overleaf...**

(c) What are the values of $t$ for when petrol will cost 202 cents per litre. **3**

$202 = 180 + 44\sin\left(\frac{2\pi t}{183}\right)$

$44\sin\left(\frac{2\pi t}{183}\right) = 22$

$\sin\left(\frac{2\pi t}{183}\right) = \frac{1}{2}$

related $\angle = \frac{\pi}{6}$

$\therefore \frac{2\pi t}{183} = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \frac{25\pi}{6}, \frac{29\pi}{6}, \ldots$ (1)

$t = 15.25, 76.25, 198.25, 259.25, 381.25, \ldots$ (1)

$(0 \le t \le 366)$

$\therefore 15.25, 76.25, 198.25, 259.25$

during 15th, 76th, 198th & 259th day. (1)

**Examination continues overleaf...**

**Question 29** (4 marks)

(a) Prove that $(1 - \sin x)(\sec x + \tan x) = \cos x$. **2**

$LHS = (1 - \sin x)(\sec x + \tan x)$

$= \sec x + \tan x - \sin x \sec x - \sin x \tan x$

$= \frac{1}{\cos x} + \frac{\sin x}{\cos x} - \sin x \times \frac{1}{\cos x} - \sin x \times \frac{\sin x}{\cos x}$ ①

$= \frac{1 + \sin x - \sin x - \sin^2 x}{\cos x}$

$= \frac{1 - \sin^2 x}{\cos x}$

$= \frac{\cos^2 x}{\cos x} = \cos x = RHS.$ ①

(b) Hence, or otherwise, evaluate $\int_0^{\frac{\pi}{2}} \sin^2 x(1 - \sin x)(\sec x + \tan x)\, dx$. **2**

$\int_0^{\frac{\pi}{2}} \sin^2 x\,(1-\sin x)(\sec x + \tan x)\, dx$

$= \int_0^{\frac{\pi}{2}} \sin^2 x \cos x \, dx$ ①

$= \left[\frac{1}{3}(\sin x)^3\right]_0^{\frac{\pi}{2}}$

$= \frac{1}{3}\left[\left(\sin \frac{\pi}{2}\right)^3 - (\sin 0)^3\right]$ ①

$= \frac{1}{3}$

**Question 30** (7 marks)

The population size $P$ of a species of birds living in a wildlife preserve increases at a rate of

$$\frac{dP}{dt} = 9e^{\frac{t^2}{5}} - 2t \quad \text{for } t \geq 0$$

where $t$ is the time in months. It is known that the initial population of the bird is 34.

(a) Use the trapezoidal rule with 4 subintervals to estimate $\int_0^4 e^{\frac{t^2}{5}}\, dt$. **3**
Hence, show that at $t = 4$, there are approximately 218 birds.

| $t$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $f(t)$ | 1 | $e^{\frac{1}{5}}$ | $e^{\frac{4}{5}}$ | $e^{\frac{9}{5}}$ | $e^{\frac{16}{5}}$ |

$\int_0^4 e^{\frac{t^2}{5}}\, dt = \frac{\frac{4-0}{2}}{4}\left(1 + 2\left(e^{\frac{1}{5}} + e^{\frac{4}{5}} + e^{\frac{9}{5}}\right) + e^{\frac{16}{5}}\right)$ (1)

$= 22.262\ldots$

$P_4 - P_0 = \int_0^4 9e^{\frac{t^2}{5}} - 2t\, dt$ (1)

$P_4 = \int_0^4 9e^{\frac{t^2}{5}} - 2t\, dt + 34$

$= 9\int_0^4 e^{\frac{t^2}{5}}\, dt - \int_0^4 2t\, dt + 34$

$= 9(22.262) - [t^2]_0^4 + 34$

$= 9(22.262) - 4 + 0 + 34$

$= 218.36\ldots$

$= 218.$ (1)

After 4 months, a coal mine was built near the wildlife preserve and pollution from the mine affects the population of the birds from $t = 4$ onwards.

Environmental modelling now reveals the population $P$ can now be modelled by the equation

$$P = Ate^{-0.05t} - 100 \qquad t \geq 4$$

(b) Using part (a), show that $A \approx 97$. **1**

at $t = 4$, $P = 218$

$218 = 4Ae^{-0.2} - 100$

$A = \frac{218 + 100}{4e^{-0.2}}$

$= 97.1015\ldots$

$= 97$

**Examination continues overleaf…**

(c) Determine the maximum population size after the mine has been built. Leave your answer correct to the nearest integer. **3**

$P(t) = 97te^{-0.05t} - 100$

$u = 97t \qquad v = e^{-0.05t}$

$u' = 97 \qquad v' = -0.05e^{-0.05t}$

$P'(t) = 97e^{-0.05t} - 97 \times 0.05te^{-0.05t}$ (1)

$= 97e^{-0.05t}(1 - 0.05t)$

when $P'(t) = 0$;

$97e^{-0.05t} = 0$ or $1 - 0.05t = 0$

but $e^{-0.05t} > 0$ $\qquad 0.05t = 1$

$t = 20.$ (1)

| $t$ | 19 | 20 | 21 |
|---|---|---|---|
| $P'(t)$ | 1.875... | 0 | -1.6979... |
| | / | — | \ |

$\therefore$ max.

$P(20) = 97(20)e^{-0.05 \times 20} - 100$

$= 613.686\ldots$

$\therefore$ max population $= 613.$ (1)

**End of paper.**

## Answer sheet for Section I

Mark answers to Section I by fully blackening the correct circle, e.g "●"

**NESA STUDENT #:** ............................................

**Class** (please ✔)

- ◯ 12MAA.1 – Mr Lam
- ◯ 12MAX.2 – Ms Lee
- ◯ 12MAX.1 – Ms C. Kim
- ◯ 12MAX.3 – Ms J. Kim

**Directions for multiple choice answers**

- Read each question and its suggested answers.
- Select the alternative Ⓐ, Ⓑ, Ⓒ, or Ⓓ that best answers the question.
- Mark only one circle per question. There is only *one* correct choice per question.
- Fill in the response circle completely, using blue or black pen, e.g.

  Ⓐ Ⓑ ● Ⓓ

- If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

  Ⓐ Ⓑ ⊗ ●

- If you continue to change your mind, write the word `correct` and clearly indicate your final choice with an arrow as shown below:

  Ⓐ Ⓑ ⊗ ⊗ ← correct

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1 –** | Ⓐ | Ⓑ | Ⓒ | Ⓓ | **6 –** | Ⓐ | Ⓑ | Ⓒ | Ⓓ |
| **2 –** | Ⓐ | Ⓑ | Ⓒ | Ⓓ | **7 –** | Ⓐ | Ⓑ | Ⓒ | Ⓓ |
| **3 –** | Ⓐ | Ⓑ | Ⓒ | Ⓓ | **8 –** | Ⓐ | Ⓑ | Ⓒ | Ⓓ |
| **4 –** | Ⓐ | Ⓑ | Ⓒ | Ⓓ | **9 –** | Ⓐ | Ⓑ | Ⓒ | Ⓓ |
| **5 –** | Ⓐ | Ⓑ | Ⓒ | Ⓓ | **10 –** | Ⓐ | Ⓑ | Ⓒ | Ⓓ |